Let us represent the scalar function being integrated as $f(x, y)$. Since $x^2 + y^2 = R^2$, we can write:

$$f(x, y) = \frac{2xy}{y + y^4 + 2x^2y^2 + x^4} = \frac{2xy}{y + (x^2 + y^2)^2} = \frac{2xy}{y + R^4}$$

Further, see that the following parametrizations of C_R differ only in direction: $\langle h(t), g(t) \rangle$ where $h(t) = R\sin(t)$ and $g(t) = R\cos(t)$ for t in $[0, 2\pi]$; and $\langle -h(t), g(t) \rangle$ for t in $[0, 2\pi]$. Since line integrals over a scalar field are independent of path direction, we have

$$\begin{aligned}
\int_{C_R} f(x, y)\,\mathrm{d}s &= \int_0^{2\pi} f(h(t), g(t)) \sqrt{\left(\frac{\mathrm{d}(h(t))}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}(g(t))}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \\
&= \int_0^{2\pi} \frac{2(R\sin(t))(R\cos(t))}{(R\cos(t)) + R^4} \sqrt{(R\cos(t))^2 + (-R\sin(t))^2}\,\mathrm{d}t \\
&= \int_0^{2\pi} \frac{2R^2\sin(t)\cos(t)}{R\cos(t) + R^4} (R)\,\mathrm{d}t \\
&= 2R^2 \int_0^{2\pi} \frac{\sin(t)\cos(t)}{\cos(t) + R^3}\,\mathrm{d}t
\end{aligned}$$

and

$$\begin{aligned}
\int_{C_R} f(x, y)\,\mathrm{d}s &= \int_0^{2\pi} f(-h(t), g(t)) \sqrt{\left(\frac{\mathrm{d}(-h(t))}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}(g(t))}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \\
&= -2R^2 \int_0^{2\pi} \frac{\sin(t)\cos(t)}{\cos(t) + R^3}\,\mathrm{d}t
\end{aligned}$$

Taking advantage of this,

$$
\begin{aligned}
2\int_{C_R} f(x,y)\,\mathrm{d}s &= \int_{C_R} f(x,y)\,\mathrm{d}s + \int_{C_R} f(x,y)\,\mathrm{d}s \\
&= \left(2R^2\int_0^{2\pi} \frac{\sin(t)\cos(t)}{\cos(t)+R^3}\,\mathrm{d}t\right) + \left(-2R^2\int_0^{2\pi} \frac{\sin(t)\cos(t)}{\cos(t)+R^3}\,\mathrm{d}t\right) \\
&= 0 \\
&\Rightarrow \int_{C_R} f(x,y)\,\mathrm{d}s = 0
\end{aligned}
$$

Thus,

$$\lim_{R\to\infty}\int_{C_R} f(x,y)\,\mathrm{d}s = \lim_{R\to\infty} 0 = 0 \qquad \square$$